

New holographic Chaplygin gas model of dark energy

M. Malekjani^{*1} and A.Khodam-Mohammadi^{†1}

¹*Department of Physics, Faculty of Science,*

Bu-Ali Sina University, Hamedan 65178, Iran

Abstract

In this work, we investigate the holographic dark energy model with new infrared cut-off (new HDE model) proposed by Granda and Oliveros. Using this new definition for infrared cut-off, we establish the correspondence between new HDE model and standard Chaplygin gas (SCG), generalized Chaplygin gas (GCG) and modified Chaplygin gas (MCG) scalar field models in non-flat universe. The potential and dynamics for these scalar field models, which describe the accelerated expansion of the universe are reconstructed. According to the evolutionary behavior of new HDE model, we derive the same form of dynamics and potential for different SCG, GCG and MCG models. We also calculate the squared sound speed of new HDE model as well as for SCG, GCG and MCG models and investigate the new HDE Chaplygin gas models from the viewpoint of linear perturbation theory. All results in non-flat universe are also discussed in the limiting case of flat universe, i.e. $k = 0$.

^{*} Email:malekjani@basu.ac.ir

[†] Email:khodam@basu.ac.ir

I. INTRODUCTION

Recent astronomical data from distant Ia supernova, Large Scale Structure (LSS) and Cosmic Microwave Background (CMB) [1] indicate that the current universe is not only expanding, but also is experiencing an accelerated expansion. The accelerated expansion can be driven by an exotic fluid with negative pressure, the so-called dark energy (DE) [2, 3]. The nature of DE is still unknown and many theoretical models have been suggested to describe its behavior. Although, the simplest theoretical candidate of DE is the cosmological constant with the equation of state independent of cosmic time, $w = -1$ [4], but it suffers the two well-known problems namely "fine-tuning" and "cosmic coincidence" [2]. Both of these problems are related to the DE density. In order to alleviate or even solve these problems, many dynamical DE models have been suggested, whose equation of state and their DE density are time-varying. It is worth noting that the predictions of cosmological constant model is still fitted to the current observation [5]. Therefore, a suggested dynamical DE model should not be faraway from the cosmological constant model. Dynamical DE models can be classified into two categories i) The scalar field DE models including quintessence [6], K-essence [7], phantoms [8], tachyon [9], dilaton [10], quintom [11] and so forth. ii) The interacting DE models, by considering the interaction between dark matter and DE, including Chaplygin gas [12], braneworld models [13], holographic DE [14] and agegraphic DE [15] models, etc.

The problem of DE has been investigated in the framework of string theory and quantum gravity. Despite the lack of a complete theory in quantum gravity, we can make some efforts to investigate the nature of DE according to the principles of quantum gravity. The holographic DE (HDE) and agegraphic DE (ADE) models have been suggested based on the principle of quantum gravity theory. The ADE model is based on the uncertainty relation of quantum mechanics together with the gravitational effect in general relativity. The HDE model is constructed based on the holographic principle [14, 16, 17]. Based on the validity of the effective local quantum field theory, a short distance (UV) cut-off Λ is related to the long distance (IR) cut-off L due to the limit set by the formation of a black hole [16]. Applying the UV-IR relationship constrains the total vacuum energy in a box of volume L^3 , where the total vacuum energy can not be greater than the mass of a black hole with the

same size. This upper limit for vacuum energy density is given by

$$\rho_\Lambda \leq M_p^2 L^{-2}, \quad (1)$$

where M_p is the reduced Planck mass and L is an IR cutoff. Saturating the inequality, we obtain the HDE density

$$\rho_\Lambda = 3c^2 M_p^2 L^{-2}, \quad (2)$$

where c is a free dimensionless constant and the numeric coefficient is chosen for convenience. The HDE model has been constrained by various astronomical observation [18–22] and also investigated widely in the literature [23–25]. The IR cutoff L is related to the large scale of the universe, for example Hubble horizon, future event horizon or particle horizon. If we take the size of Hubble horizon or particle horizon as a length scale L , the accelerated expansion of the universe can not be derived by HDE model [14]. However, in the case of event horizon, HDE model can derive the universe with accelerated expansion [14]. The arising problem with the event horizon is that it is a global concept of spacetime and existence of it depends on the future evolution of the universe only for a universe with forever accelerated expansion. Furthermore, the HDE with the event horizon as a length scale is not compatible with the age of some old high redshift objects [26].

Granda and Oliveros (GO, here after) proposed a new IR cut-off containing the local quantities of Hubble and time derivative Hubble scales [27]. The advantages of HDE with GO cutoff (new HDE model) is that it depends on local quantities and avoids the causality problem appearing with event horizon IR cutoff. The new HDE model can also obtain the accelerated expansion of the universe [27]. GO showed that the transition redshift from deceleration phase ($q > 0$) to acceleration phase ($q < 0$) is consistent with current observational data [27, 28]. The new HDE model has been extended into the scalar field models both in flat and non-flat universe [29]. The correspondence between this model and scalar fields allows us to reconstruct the potentials and the dynamics of scalar fields. The new HDE model has also been investigated in non-flat universe [30].

However, the early inflation era leads to a flat universe, but the effect of curvature can not be neglected at present time. Observationally, the CMB experiments preferred a closed universe with small positive curvature [31]. The WMAP analysis also provides further confidence to show that a closed universe with positively curved space is marginally preferred [32]. Therefore, we are motivated to investigate the new HDE model in a non flat universe.

On the other hand, the Chaplygin gas is one of the candidate of DE models to explain the accelerated expansion of the universe. The striking features of Chaplygin gas DE is that it can be assumed as a possible unification of dark matter and DE. The Chaplygin gas plays a dual role at different epoch of the history of the universe: it can be as a dust-like matter in the early time, and as a cosmological constant at late times. This model from the field theory points of view is investigated in [33]. The Chaplygin gas emerges as an effective fluid associated with D-branes [34] and can also be obtained from the Born-Infeld action [35]. The simplest form of Chaplygin gas model called standard Chaplygin gas (SCG) which has been used to explain the accelerated expansion of universe [36]. Although the SCG model can interpret the accelerated expansion of universe, but it can not explain the astrophysical problems such as structure formation and cosmological perturbation power spectrum [37]. Subsequently, the SCG is extended into the generalized Chaplygin gas (GCG) which can construct viable cosmological models. The GCG model is also modified into the modified Chaplygin gas (MCG) which can show the radiation era in the early universe [?]. The correspondence between the HDE and ADE models with the Chaplygin gas energy density has been established in [38].

It should be noted that the correspondence between HDE and ADE with SCG is problematic. This problem arises from the viewpoint of linear perturbation theory. In this theory, the SCG model is stable against small perturbation [39], While the HDE and ADE have an instability of a given perturbation.[40]. The crucial quantity which shows the stability or instability of density perturbation is squared speed of sound. For positive value of this quantity, we have a regular propagating mode (stability), while negative value of squared speed shows an exponentially growing mode (instability) for a density perturbation.

Here, in this work, we establish the connection between the SCG, GCG and MCG models with new HDE model in non-flat universe. This connection allows us to reconstruct the potentials and the dynamics of the scalar fields according to evolutionary form of new HDE to describe the SCG, GCG and MCG cosmology. We also calculate the squared speed for new HDE model and discuss the holographic interpretation of SCG, GCG and MCG models from the viewpoint of linear perturbation theory. The results are also discussed in the limiting case of flat universe.

II. NEW HDE MODEL

The energy density of new HDE model is [27]

$$\rho_\Lambda = 3M_P^2(\alpha H^2 + \beta \dot{H}), \quad (3)$$

where α and β are constants, H is the Hubble parameter and dot denotes the time derivative with respect to the cosmic time. The Friedmann-Robertson-Walker (FRW) metric for a universe with spatial curvature k is

$$ds^2 = dt^2 - a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), \quad (4)$$

where $a(t)$ is the scale factor, and $k = -1, 0, 1$ represents the open, flat, and closed universes, respectively. A closed universe with small positive curvature ($\Omega_k \sim 0.02$) is compatible with observation [42]. Like [29], we restrict ourselves to the current DE dominated universe. Hence, the Freidmann equation is written as

$$H^2 + \frac{k}{a^2} = \frac{1}{3M_p^2} \rho_\Lambda. \quad (5)$$

Substituting Eq.(3) in (5) yields

$$\frac{dH^2}{dx} + \frac{2}{\beta}(\alpha - 1)H^2 = \frac{2}{\beta}ke^{-2x}, \quad (6)$$

where $x = \ln a$. Integrating Eq.(6) with respect to x gives the following relation for Hubble parameter in new HDE dominated non-flat universe

$$H^2 = \frac{k}{\alpha - \beta - 1}e^{-2x} + \gamma e^{-\frac{2}{\beta}(\alpha-1)x}, \quad (7)$$

where γ is an integration constant. From conservation equation

$$\dot{\rho}_\Lambda + 3H(1 + \omega_\Lambda)\rho_\Lambda = 0, \quad (8)$$

and using Eq.(3), one can easily obtain the equation of state (EoS) parameter, $\omega_\Lambda = p_\Lambda/\rho_\Lambda$, as

$$\omega_\Lambda = -1 - \frac{2\alpha H\dot{H} + \beta\ddot{H}}{3H(\alpha H^2 + \beta\dot{H})} \quad (9)$$

Inserting H from Eq.(7) in (9) obtains

$$\omega_\Lambda = -\frac{1}{3} \left(\frac{k \left(\frac{\alpha-\beta}{\alpha-\beta-1} \right) e^{-2x} + \gamma \left(\frac{3\beta-2\alpha+2}{\beta} \right) e^{-\frac{2}{\beta}(\alpha-1)x}}{k \left(\frac{\alpha-\beta}{\alpha-\beta-1} \right) e^{-2x} + \gamma e^{-\frac{2}{\beta}(\alpha-1)x}} \right), \quad (10)$$

which expresses the time-dependent EoS parameter of new HDE in non-flat universe. The time dependence of EoS parameter of DE allows it to transit from $w_\Lambda > -1$ to $w_\Lambda < -1$ [46]. The analysis of DE models from the observational point of view indicates that a model with w_Λ crossing -1 in the near past is favored [44].

Putting $k = 0$ in Eqs. (7) and (10), the Hubble parameter and EoS parameter of new HDE model in flat universe are reduced as

$$H^2 = \gamma e^{-\frac{2}{\beta}(\alpha-1)x} \quad (11)$$

$$\omega_\Lambda = -1 + \frac{2}{3} \frac{\alpha - 1}{\beta}, \quad (12)$$

Contrary to the non flat universe, the EoS parameter in flat universe is constant with cosmic time. In order to obtain the accelerated expansion of universe, $-1 < w_\Lambda < -1/3$, the constants α and β must be limited as: $\beta > \alpha - 1$ if $\alpha > 1$ or $\beta < \alpha - 1$ if $\alpha < 1$. Also, the new HDE can achieve the phantom phase, $w_\Lambda < -1$, for $\alpha < 1$, $\beta > 0$ or $\alpha > 1$, $\beta < 0$ [29].

III. NEW HDE AND STANDARD CHAPLYGIN GAS

The equation of state of a perfect fluid, standard Chaplygin gas (SCG), is given by

$$p_D = \frac{-A}{\rho_D}, \quad (13)$$

where A is a positive constant, P_D and ρ_D are the pressure and energy density, respectively. Substituting the equation of state of SCG (i.e., Eq. 13) into the relativistic energy conservation equation, leads to evolving density as

$$\rho_D = \sqrt{A + \frac{B}{a^6}}. \quad (14)$$

where B is an integration constant. The energy density and pressure of the scalar field, regarding the SCG dark energy is written as

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) = \sqrt{A + \frac{B}{a^6}}, \quad (15)$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) = \frac{-A}{\sqrt{A + \frac{B}{a^6}}}, \quad (16)$$

Hence, it is easy to obtain the scalar potential and the kinetic energy terms for the SCG model as

$$V(\phi) = \frac{2Aa^6 + B}{2a^6 \sqrt{A + \frac{B}{a^6}}}, \quad (17)$$

$$\dot{\phi}^2 = \frac{B}{a^6 \sqrt{A + \frac{B}{a^6}}}. \quad (18)$$

In this section, first we establish the correspondence between new HDE and SCG model and re-construct the potential and the dynamics of scalar fields in non-flat universe, then we discuss the limiting case of flat universe. Assuming non-flat universe, by equating the energy density of SCG, Eq.(14), and energy density of new HDE, Eq. (3), the constant B can be obtained as

$$B = a^6 \left(-A + 9M_p^4 \left(\frac{\alpha - \beta}{\alpha - \beta - 1} k e^{-2x} + \gamma e^{-\frac{2}{\beta}(\alpha-1)x} \right)^2 \right). \quad (19)$$

Using Eqs.(13), (10) and (14), we have

$$\omega_\Lambda = \frac{p_d}{\rho_d} = \frac{-A}{A + B a^{-6}} = -\frac{1}{3} \left(\frac{k \left(\frac{\alpha - \beta}{\alpha - \beta - 1} \right) e^{-2x} + \gamma \left(\frac{3\beta - 2\alpha + 2}{\beta} \right) e^{-\frac{2}{\beta}(\alpha-1)x}}{k \left(\frac{\alpha - \beta}{\alpha - \beta - 1} \right) e^{-2x} + \gamma e^{-\frac{2}{\beta}(\alpha-1)x}} \right), \quad (20)$$

Inserting Eq.(19) in (20), one can obtain the constant A as

$$A = 9M_p^4 \left(\frac{\alpha - \beta}{\alpha - \beta - 1} k e^{-2x} + \frac{3\beta - 2\alpha + 1}{\beta} \gamma e^{-\frac{2}{\beta}(\alpha-1)x} \right) \left(\frac{\alpha - \beta}{\alpha - \beta - 1} k e^{-2x} + \gamma e^{-\frac{2}{\beta}(\alpha-1)x} \right) \quad (21)$$

Substituting A and B in Eqs.(17) and (18), we can rewrite the scalar potential term as

$$V(\phi) = M_p^2 \left(\frac{2(\alpha - \beta)}{\alpha - \beta - 1} k e^{-2x} + \frac{3\beta - \alpha + 1}{\beta} \gamma e^{-\frac{2}{\beta}(\alpha-1)x} \right) \quad (22)$$

and kinetic energy term as

$$\dot{\phi} = M_p \sqrt{\left(\frac{2(\alpha - \beta)}{\alpha - \beta - 1} k e^{-2x} + \frac{2(\alpha - 1)}{\beta} \gamma e^{-\frac{2}{\beta}(\alpha-1)x} \right)} \quad (23)$$

Using $\dot{\phi} = \phi' H$, where prime denotes the derivative with respect to $x = \ln a$, we have

$$\phi' = M_p \sqrt{\frac{\left(\frac{2(\alpha - \beta)}{\alpha - \beta - 1} k e^{-2x} + \frac{2(\alpha - 1)}{\beta} \gamma e^{-\frac{2}{\beta}(\alpha-1)x} \right)}{\frac{1}{\alpha - \beta - 1} k e^{-2x} + \gamma e^{-\frac{2}{\beta}(\alpha-1)x}}} \quad (24)$$

Integrating Eq.(24), one can obtain the evolutionary treatment of scalar field as

$$\phi(a) - \phi(0) = M_p \int_0^x \sqrt{\frac{\left(\frac{2(\alpha-\beta)}{\alpha-\beta-1}ke^{-2x} + \frac{2(\alpha-1)}{\beta}\gamma e^{-\frac{2}{\beta}(\alpha-1)x}\right)}{\frac{1}{\alpha-\beta-1}ke^{-2x} + \gamma e^{-\frac{2}{\beta}(\alpha-1)x}}} dx \quad (25)$$

where we take $\ln a_0 = 0$ for present time. Here we established the connection between new HDE and SCG models and reconstructed the potential and the dynamics of new HDE model to describe the non-flat SCG cosmology.

In the limiting case of flat universe, putting $k = 0$ in Eqs.(19) and (21), the constants A and B of SCG, are reduced as

$$B = a^6 \left(-A + 9M_p^4 \gamma^2 e^{-\frac{4}{\beta}(\alpha-1)x} \right) \quad (26)$$

$$A = 9M_p^4 \frac{3\beta - 2\alpha + 1}{\beta} \gamma^2 e^{-\frac{4}{\beta}(\alpha-1)x} \quad (27)$$

The scalar potential and kinetic energy terms are reduced to

$$V(\phi) = M_p^2 \frac{3\beta - \alpha + 1}{\beta} \gamma e^{-\frac{2x(\alpha-1)}{\beta}} \quad (28)$$

$$\dot{\phi} = M_p \sqrt{\frac{2(\alpha-1)}{\beta} \gamma e^{-\frac{2}{\beta}(\alpha-1)x}} \quad (29)$$

Using $\dot{\phi} = \phi' H$, where prime denotes the derivative with respect to $x = \ln a$, we have

$$\phi' = M_p \sqrt{\frac{2(\alpha-1)}{\beta}} \quad (30)$$

The evolutionary form of scalar field can be obtained by integrating of Eq.(30) as follows

$$\phi(a) - \phi(0) = M_p \int_0^x \sqrt{\frac{2(\alpha-1)}{\beta}} dx = M_p \sqrt{\frac{2(\alpha-1)}{\beta}} \ln a \quad (31)$$

IV. NEW HDE AND GENERALIZED CHAPLYGIN GAS

Although, the SCG model can interpret the accelerated expansion of universe, but it does not solve the cosmological problems like structure formation and cosmological perturbation power spectrum [37]. Subsequently, the SCG was modified to the following form [35]

$$p_D = \frac{-A}{\rho_D^\eta}, \quad (32)$$

called generalized Chaplygin gas (GCG). Two free parameters involved in GCG: one is A and the other η . Similar with SCG, The GCG fluid behaves like dust for small size of the universe while it acts as cosmological constant when universe gets sufficiently large. Using the energy conservation equation for GCG, the evolving energy density is obtained as

$$\rho_D = (A + Ba^{-3\delta})^{\frac{1}{\delta}} \quad (33)$$

where $\delta = \eta + 1$. Regarding the scalar field model, the energy density and pressure of GCG is given by

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) = (A + Ba^{-3\delta})^{-\delta}, \quad (34)$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) = -A(A + B^{-3\delta})^{-\frac{\delta-1}{\delta}}, \quad (35)$$

Hence, the scalar potential and kinetic energy term can be obtained as

$$V(\phi) = \frac{2A + Ba^{-3\delta}}{2(Ba^{-3\delta})^{\frac{\delta-1}{\delta}}}, \quad (36)$$

$$\dot{\phi}^2 = \frac{Ba^{-3\delta}}{(A + Ba^{-3\delta})^{\frac{\delta-1}{\delta}}}. \quad (37)$$

In this section we establish the connection between new HDE and GCG models and re-construct the scalar field DE model. First we assume a general non-flat universe, then we discuss the limiting case of flat universe. In non-flat case, by equating Eq.(3) with (33), the constant B can be obtained as

$$B = a^{3\delta} \left(-A + (3M_p^2)^\delta \left(\frac{\alpha - \beta}{\alpha - \beta - 1} k e^{-2x} + \gamma e^{-\frac{2}{\beta}(\alpha-1)x} \right)^\delta \right) \quad (38)$$

Using Eqs.(32), (33) and (10), we have

$$\omega_\Lambda = \frac{p_d}{\rho_d} = \frac{-A}{(A + Ba^{-3\delta})} = -\frac{1}{3} \left(\frac{k \left(\frac{\alpha - \beta}{\alpha - \beta - 1} \right) e^{-2x} + \gamma \left(\frac{3\beta - 2\alpha + 2}{\beta} \right) e^{-\frac{2}{\beta}(\alpha-1)x}}{k \left(\frac{\alpha - \beta}{\alpha - \beta - 1} \right) e^{-2x} + \gamma e^{-\frac{2}{\beta}(\alpha-1)x}} \right), \quad (39)$$

Inserting Eq.(38) in (39), the constant A is obtained as

$$A = (3M_p^2)^\delta \left(\frac{\alpha - \beta}{\alpha - \beta - 1} k e^{-2x} + \gamma e^{-\frac{2}{\beta}(\alpha-1)x} \right)^{\delta-1} \left(\frac{\alpha - \beta}{\alpha - \beta - 1} k e^{-2x} + \frac{3\beta - 2\alpha + 2}{\alpha - \beta - 1} \gamma e^{-\frac{2}{\beta}(\alpha-1)x} \right) \quad (40)$$

Substituting Eqs.(38) and (40) in Eqs.(36) and (37), using Eq.(7), the potential and dynamics of new HDE generalized Chaplygin gas is obtained as follows

$$V(\phi) = M_p^2 \left(\frac{2(\alpha - \beta)}{\alpha - \beta - 1} k e^{-2x} + \frac{3\beta - \alpha + 1}{\beta} \gamma e^{-\frac{2}{\beta}(\alpha-1)x} \right) \quad (41)$$

$$\dot{\phi} = M_p \sqrt{\left(\frac{2(\alpha - \beta)}{\alpha - \beta - 1} k e^{-2x} + \frac{2(\alpha - 1)}{\beta} \gamma e^{-\frac{2}{\beta}(\alpha - 1)x} \right)} \quad (42)$$

which are exactly same as Eqs.(22) and (23) for new HDE-SCG model. Therefore, the potential and the dynamics of new HDE-GCG coincide with new HDE-SCG model. Following the same steps as done for the SCG model, the evolutionary form of scalar field describing the new HDE-GCG is as follows

$$\phi(a) - \phi(0) = M_p \int_0^x \sqrt{\frac{\left(\frac{2(\alpha - \beta)}{\alpha - \beta - 1} k e^{-2x} + \frac{2(\alpha - 1)}{\beta} \gamma e^{-\frac{2}{\beta}(\alpha - 1)x} \right)}{\frac{1}{\alpha - \beta - 1} k e^{-2x} + \gamma e^{-\frac{2}{\beta}(\alpha - 1)x}}} dx \quad (43)$$

The potential and the dynamics of new HDE model which describe the GCG cosmology is similar with SCG cosmology.

In the limiting case of flat universe, the Hubble parameter and EoS parameter of new HDE are given by Eqs. (11) and (12). Using Eq.(11) and putting $k = 0$ in Eqs. (38) and (40), the constants B and A for GCG in flat universe are reduced as follows

$$B = a^{3\delta} \left(-A + (3M_p^2 \gamma e^{-\frac{2}{\beta}(\alpha - 1)x})^\delta \right) \quad (44)$$

$$A = \frac{3\beta - 2\alpha + 2}{\beta} \left(3M_p^2 \gamma e^{-\frac{2}{\beta}(\alpha - 1)x} \right)^\delta \quad (45)$$

Same as non-flat case, the scalar potential and the kinetic energy terms of new HDE, describing the flat GCG cosmology, which can be obtained by putting $k = 0$ in Eqs. (41) and (42), are same as Eq. (28) and (29) for new HDE-SCG in flat universe. Therefore, the evolutionary form of scalar field describing the new HDE-GCG in flat universe is given by Eq. (31).

V. NEW HDE AND MODIFIED CHAPLYGIN GAS

After the GCG was introduced, the new model of Chaplygin gas which is called modified Chaplygin gas (MCG) was proposed [45]. The equation of state of MCG is written as

$$p_D = B\rho_D - \frac{A_0}{\rho_D^\eta} \quad (46)$$

where B and A_0 are constant parameters and $0 \leq \eta \leq 1$. An interesting feature of MCG is that it can show the radiation era in the early universe. At the late time, the MCG behaves

as cosmological constant and can be fitted to Λ CDM model. The equation of state and evolving energy density of MCG is given by

$$p_D = B\rho_D - \frac{A_0 a}{\rho_D^\eta} = \frac{1}{2}\dot{\phi}^2 - V(\phi), \quad (47)$$

$$\rho_D = \left[\frac{3\delta A_0}{[3\delta(B+1)]} - \frac{C}{a^{3\delta(B+1)}} \right]^{\frac{1}{\delta}} = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad (48)$$

where $\delta = \eta + 1$, a is a scale factor and B , A_0 , C are constants.

From Eqs. (47) and (48), the kinetic and potential terms of MCG can be obtained as

$$\begin{aligned} \dot{\phi}^2 = & (1+B) \left[\frac{3\delta A_0}{[3\delta(B+1)]} - \frac{C}{a^{3\delta(B+1)}} \right]^{\frac{1}{\delta}} \\ & - \frac{A_0}{\left[\frac{3\delta A_0}{[3\delta(B+1)]} - \frac{C}{a^{3\delta(B+1)}} \right]^{\frac{\delta-1}{\delta}}}. \end{aligned} \quad (49)$$

$$\begin{aligned} V(\phi) = & \frac{(1-B)}{2} \left[\frac{3\delta A_0}{[3\delta(B+1)]} - \frac{C}{a^{3\delta(B+1)}} \right]^{\frac{1}{\delta}} \\ & + \frac{A_0}{2 \left[\frac{3\delta A_0}{[3\delta(B+1)]} - \frac{C}{a^{3\delta(B+1)}} \right]^{\frac{\delta-1}{\delta}}}. \end{aligned} \quad (50)$$

We now establish the correspondence between new HDE and MCG model and reconstruct the potential and the dynamics of the scalar field in the presence of new HDE.

In non-flat case, by equating Eqs. (48) and (3), we get

$$C = a^{3\delta(B+1)} \left(\frac{3\delta A_0}{[3\delta(B+1)]} - \left[3M_P^2 \left(\frac{\alpha - \beta}{\alpha - \beta - 1} k e^{-2x} + \gamma e^{-\frac{2}{\beta}(\alpha-1)x} \right) \right]^\delta \right). \quad (51)$$

Using Eqs. (47) and (48), the EoS parameter of MCG is

$$w_D = \frac{p_D}{\rho_D} = B - \frac{A_0}{\rho_D^{\eta+1}} \quad (52)$$

Equating $w_D = w_\Lambda$ and replacing the energy densities of new HDE and MCG, $\rho_D = \rho_\Lambda$ in Eq.(52), we have

$$w_\Lambda = B - \frac{A_0}{(3M_P^2[\alpha H^2 + \beta \dot{H}])^\delta} = -\frac{1}{3} \left(\frac{k \left(\frac{\alpha - \beta}{\alpha - \beta - 1} \right) e^{-2x} + \gamma \left(\frac{3\beta - 2\alpha + 2}{\beta} \right) e^{-\frac{2}{\beta}(\alpha-1)x}}{k \left(\frac{\alpha - \beta}{\alpha - \beta - 1} \right) e^{-2x} + \gamma e^{-\frac{2}{\beta}(\alpha-1)x}} \right) \quad (53)$$

Therefore, the above equation obtains the value of the parameter B_0 as

$$A_0 = \left(3M_P^2 \left(\frac{\alpha - \beta}{\alpha - \beta - 1} k e^{-2x} + \gamma e^{-\frac{2}{\beta}(\alpha-1)x} \right) \right)^\delta \left(B + \frac{1}{3} \frac{k \left(\frac{\alpha - \beta}{\alpha - \beta - 1} \right) e^{-2x} + \gamma \left(\frac{3\beta - 2\alpha + 2}{\beta} \right) e^{-\frac{2}{\beta}(\alpha-1)x}}{k \left(\frac{\alpha - \beta}{\alpha - \beta - 1} \right) e^{-2x} + \gamma e^{-\frac{2}{\beta}(\alpha-1)x}} \right) \quad (54)$$

Substituting Eqs. (51) and (54) in Eqs. (49) and (50), we can re-write the scalar potential and kinetic energy terms as

$$V(\phi) = M_P^2 \left(\frac{2(\alpha - \beta)}{\alpha - \beta - 1} k e^{-2x} + \frac{3\beta - \alpha + 1}{\beta} \gamma e^{-\frac{2}{\beta}(\alpha-1)x} \right) \quad (55)$$

$$\dot{\phi} = M_P \sqrt{\left(\frac{2(\alpha - \beta)}{\alpha - \beta - 1} k e^{-2x} + \frac{2(\alpha - 1)}{\beta} \gamma e^{-\frac{2}{\beta}(\alpha-1)x} \right)} \quad (56)$$

It is interesting to note that the above potential and kinetic energy expressions for the new HDE-MCG model coincide with those obtained for new HDE-SCG and new HDE-GCG models. Hence, like previous models, the evolutionary form of scalar field describing the MCG cosmology is given by

$$\phi(a) - \phi(0) = M_P \int_0^x \sqrt{\frac{\left(\frac{2(\alpha - \beta)}{\alpha - \beta - 1} k e^{-2x} + \frac{2(\alpha - 1)}{\beta} \gamma e^{-\frac{2}{\beta}(\alpha-1)x} \right)}{\frac{1}{\alpha - \beta - 1} k e^{-2x} + \gamma e^{-\frac{2}{\beta}(\alpha-1)x}}} dx} \quad (57)$$

In flat case, same as previous sections, the Hubble parameter and EoS parameter for new HDE model is given by Eqs. (11) and (12) respectively. Putting $k = 0$ in Eqs. (51) and (54), the constants C and B_0 for MCG are reduced as

$$C = a^{3\delta(B+1)} \left(\frac{3\delta A_0}{[3\delta(B+1)]} - \left[3M_P^2 \gamma e^{-\frac{2}{\beta}(\alpha-1)x} \right]^\delta \right). \quad (58)$$

$$A_0 = \left(3M_P^2 \gamma e^{-\frac{2}{\beta}(\alpha-1)x} \right)^\delta \left(B + \frac{3\beta - 2\alpha + 2}{3\beta} \right) \quad (59)$$

Subsequently, the potential and kinetic energy terms are reduced as to Eq. (28) and (29). Eventually, same as new HDE-SCG and new HDE-GCG models, the evolutionary form of scalar field describing the new HDE-MCG model in flat universe is given by Eq. (31).

VI. SQUARED SPEED FOR NEW HDE AND SCG, GCG, MCG MODELS

One of the crucial physical quantities in the theory of linear perturbation is the squared speed of sound, v^2 . The sign of v^2 is important for determining the stability or instability

of a given perturbed mode. The positive sign (real value of speed) indicates the periodic propagating mode for a density perturbation and in this case we encounter with the stability for a given mode. The negative sign (imaginary value of speed) shows an exponentially growing mode for a density perturbation, means the instability for a given mode [40].

Generally, the evolution of sound speed in the linear regime of perturbation is dependent on the dynamics of background cosmology as follows [41]

$$v^2 = \frac{dp}{d\rho} = \frac{1}{3H} \frac{d}{dH} \left[H^2 \left(q - \frac{1}{2} \right) \right], \quad (60)$$

where q is the deceleration parameter. Therefore, the sign of v^2 is linked to the sign of q and to the transition epoch from CDM dominated phase to DE dominated phase. From the above description, one can easily find that the growth of perturbation in linear theory is dependent on the choice of DE model of background dynamics.

In this section, we calculate v^2 for new HDE model as well as for SCG, GCG and MCG models. The squared speed is introduced as

$$v^2 = \frac{dp}{d\rho} = \frac{\dot{p}}{\dot{\rho}} \quad (61)$$

For SCG model, using Eq.(13) and $p_D = w_D \rho_D$, we have

$$v^2 = \frac{A}{\rho^2} = -w_D \quad (62)$$

Assuming the SCG as a quintessence DE model with $-1 < w_D < 0$, the squared speed is positive. Thus, the SCG is stable against density perturbation at any cosmic scale factor.

In the case of GCG model, by using Eq.(32), v^2 is obtained as

$$v^2 = -\eta w_D \quad (63)$$

In the range of quintessence model $-1 < w_D < 0$ for GCG model, we see that the GCG model is instable against the density perturbation ($v^2 < 0$) if $\eta < 0$ and stable ($v^2 > 0$) if $\eta > 0$. In MCG model, by using Eq.(46), and considering the range of EoS of MCG as $-1 < w_D < 0$, v^2 can be obtained as

$$v^2 = -w_D + 2B = |w_D| + 2B \quad (64)$$

Therefore, the sing of v^2 is positive (stability) if $B \geq 0$. For $B < 0$, the sign of sound speed can be positive and also can be negative. In this case, v^2 is positive (stability) if $|w_D| < 2|B|$

and it is negative (instability) if $|w_D| > 2|B|$.

We now calculate the squared sound speed for new HDE model and compare it with SCG, GCG and MCG models. First, we assume a general non-flat case. Differentiating the equation of state, $p = w\rho$ with respect to time, we have

$$\dot{p} = \dot{w}_\Lambda \rho + w_\Lambda \dot{\rho} \quad (65)$$

Inserting Eq. (65) in right hand side of Eq.(61), and using Eq.(8), the squared sound speed for new HDE model is obtained as follows

$$v^2 = w_\Lambda - \frac{\dot{w}_\Lambda}{3H(1 + w_\Lambda)} \quad (66)$$

Inserting Eq.(9) in (66) and using Eq.(7), the squared sound speed in non-flat universe is obtained as

$$v^2 = -\frac{1}{3} \frac{(\alpha - 1)(-2\alpha + 3\beta + 2)H^2 + ke^{-2x}[\beta(\beta - 1) + 2\alpha - 2]}{\beta[(\alpha - 1)H^2 + ke^{-2x}(\beta - 1)]} \quad (67)$$

Let us consider the special case $k = 1$, the squared sound speed at present time is reduced as follows:

$$v^2 = -\frac{1}{3} \frac{(\alpha - 1)(-2\alpha + 3\beta + 2)(\frac{1}{\alpha - \beta - 1} + \gamma) + [\beta(\beta - 1) + 2\alpha - 2]}{\beta[\frac{\alpha - 1}{\alpha - \beta - 1} + \gamma(\alpha - 1) + (\beta - 1)]} \quad (68)$$

Adopting the best fit values: $\alpha = 0.8824$ and $\beta = 0.5016$, obtained by Y. Wang and L. Xu in non-flat universe [46], one can easily see that v^2 in Eq.(68) is positive for $\gamma < -0.7560$ and negative when $\gamma > -0.7560$. Therefore, at the present time, new HDE model is stable provided that $\gamma < -0.7560$ and instable when $\gamma > -0.7560$.

Here we discuss the validity of the correspondence between new HDE and different Chaplygin gas models which are driven in previous sections. For $\gamma < -0.7560$, new HDE is stable and on the other hand the SCG, GCG with $\eta > 0$ and also MCG with $B \geq 0$ or $(B < 0, |w_D| > 2|B|)$ are also stable against density perturbation. Therefore, in this case, a correspondence between new HDE with SCG, GCG and MCG is not problematic and the reconstructed potential and dynamics of scalar field for new HDE model which describe the SCG, GCG and MCG cosmology are logical. However, in this case, one cannot encounter with the exponentially growing mode of the perturbations. The new HDE is instable for $\gamma > -0.7560$, Also the GCG model with $\eta < 0$ and MCG with $(B < 0, |w_D| < 2|B|)$ are also instable. Hence, the correspondence between new HDE with GCG and MCG models are logical. In

this case, the correspondence between new HDE and SCG is problematic, since the SCG has a stability against perturbation. In this case, the potential and the dynamics of scalar field constructed from the evolutionary form of new HDE cannot describe the SCG cosmology. In the limiting case of flat universe, by putting $k = 0$ in Eq.(67), the squared sound speed for new HDE model is obtained as

$$v^2 = -1 + \frac{2}{3} \frac{\alpha - 1}{\beta}, \quad (69)$$

which is same as Eq.(12). Therefore, in flat case, we have $v^2 = w_\Lambda$. Adopting the best fit values: $\alpha = 0.8502$ and $\beta = 0.4817$, obtained by Y. Wang and L. Xu in flat universe [46], We can see that v^2 in Eq.(69) is obtained as -1.21 . Hence, the new HDE model in flat universe is instable at any scale factor. In flat universe, the correspondence between new HDE and GCG with $\eta < 0$ and MCG with $(B < 0, |w_D| < 2|B|)$ are logical. While the correspondence between new HDE and SCG, GCG with $\eta > 0$ and MCG with $(B \geq 0$ or $B < 0, |w_D| > 2|B|)$ is problematic. From the viewpoint of linear perturbation theory, the reconstructed potential and dynamics of scalar field for new HDE model can describe the Chaplygin cosmology, if both of DE models have a same sign of squared sound speed. Otherwise, the correspondence between them is problematic.

VII. CONCLUSION

In the context of new HDE model, the event horizon IR cut-off is replaced by new IR cut-off containing the local quantities of Hubble and time derivative Hubble scales [27]. The new HDE model not only gives the accelerated expansion of the universe, but also avoids the causality problem appearing with event horizon IR cut-off. On the other hand, among the several candidates for DE, we consider the Chaplygin gas model which unifies DE and dark matter. In this work, we established a correspondence between the new HDE density and various models of Chaplygin gas scalar field models of DE in non-flat universe. The non-flatness of universe with small positive curvature $\Omega_k \sim 0.02$ is favored by recent experimental data [31, 32]. We adopted the viewpoint that the scalar field models of DE are effective theories of an underlying theory of DE. Thus, we should be capable of using the scalar field models to mimic the evolving behavior of the new HDE and reconstructing these scalar field models. We reconstructed the potential and the dynamics of scalar field

for new HDE model which describe the SCG, GCG and MCG cosmology. In the limiting case of flat universe, we obtained the simple analytical solution for the evolutionary form of the SCG, GCG and MCG scalar fields. We also concluded that, according to the evolutionary behavior of the new HDE model, the potential and the dynamics of MCG scalar field are same as the potential and the dynamics of GCG and MCG models. Finally, from the viewpoint of linear perturbation theory, we studied the correspondence between new HDE with SCG, GCG and MCG models. We showed that the reconstructed potential and dynamics of scalar field for new HDE model can describe the Chaplygin cosmology, if both the new HDE and Chaplygin gas DE models have a same sign of squared sound speed.

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